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# Adaptive control system for an autonomous quadrotor unmanned aerial vehicle

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## Abstract

The need for a realistic adaptive controller system to operate a quadrotor unmanned aerial vehicle in potentially constrained environments has raised many interesting research questions. Flying a quadrotor UAV over a certain area requires a controller system that has the ability to adjust its parameters based on the online measurements. Under this condition, this paper proposes a new adaptive control system to stabilize a quadrotor that is designed to perform a variety of specific applications. The kinematics and dynamics of the quadrotor are derived, and then the baseline controller is redesigned in the case where there is an uncertainty in finding targets in search and rescue missions. Moreover, the control parameters are determined based on the desired performance set by the designer. To accommodate any uncertainties during the task phases, a new term is added to the feedback linearization controller in order to guarantee that the vehicle will not miss the target location. The Linear Quadratic Regulator (LQR) is used in this work as the kernel of the controller system. This method works by minimizing a cost function that allows the observer to locate targets accurately. We present the results of simulations to demonstrate the suitability of the proposed control system for such missions. The findings indicate that the proposed adaptive control system is able to accommodate any uncertainties during the task phases.

Keywords: Linear Quadratic Regulator (LQR), Adaptive Controller System, Quadrotor Unmanned Aerial Vehicle.

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# 1. Introduction

As shown in Fig. 1, quadrotor is a small aircraft whose lift is generated by four rotors [1]: front, back, right and left. Front and back are located on one axis and rotating clockwise while left and right are located on the second

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axis and rotated counterclockwise. Two frames are shown in the figure, the earth frame (*Xe, Ye, Ze*) and the body frame (*Xb, Yb, Zb*). The first frame is used to specify the location of the quadrotor while the second one is used along with the roll, pitch, and yaw to specify the quadrotor orientation. Quadrotors are emerged nowadays as a popular unmanned aerial vehicle platform. It can be controlled by the rotational speed of the rotors, and it has the potential to take-off and land vertically [2], [3], [4]. Quadrotors have the advantage that they can move in six degree of freedom, but on the other hand, they are intrinsically unstable.

To control such system, several control algorithms were investigated to stabilize the vehicle [5], [6], [7], [8]. As an example, PID control algorithm, control algorithms that use adaptive techniques for uncertainties and unmodeled dynamics, control algorithms that are based on Linear Quadratic Regulator (LQR), control algorithms that are done with backstepping, control algorithm that are based on visual feedback and finally control algorithms that are done with fuzzy techniques or neural networks. Among those, the linear quadratic regulator algorithm is a new method of controlling that is concerned with the operating of dynamic system at minimum cost. Because of their simplicity and ease of use, LQR have become very popular and become the most widely accepted method of determining optimal control policy [9]. As we will see in the next section, LQR does not compare the output to the reference but it used to find the optimal control matrices (Performance index matrix(R) and state-cost matrix (Q)) that result in some balance between system error and control effort. Adaptive control is a control method used to accommodate any parameters that are unknown or changing. This technique is differing from other controllers in that it does not need a priori information about the uncertain parameters. Least squares estimation, dynamic inversion with neural networks, and model reference adaptive control are all different types of adaptive control. Least squares estimation utilizes the outputs of the system and estimates the plant parameters in order to minimize the error while in the dynamic inversion, the error dynamics between the reference input and plant output are inverted to simplify regulator design. In model reference control, the output of a plant is compared against the output of a model that is being driven by a reference signal. The error between the model output and plant output is then used to drive the plant to the desired reference input.

As we know, quadrotor is subjected to uncertainties that are associated with external factors; those uncertainties offer a challenging control problem when designing control algorithms to such unstable vehicles. For this purpose, adaptive control system for an autonomous quadrotor unmanned aerial vehicle is presented in this paper. The remainder of this paper is organized as follows. Section 2 presents a brief survey of control algorithms developed for quadrotor vehicles. In section 3, the kinematic and dynamic model of a quadrotor is derived. In Section 4, we briefly describe the LQR controller and explain the design and tuning of this controller and the adaptive one. In section 5, we present the simulation results that demonstrate the proposed control scheme. Finally, we summarize the main results with a discussion of future works in Section 6.



Fig.1. Quadrotor schematic, showing earth and body frames.

Fig.2. Rotation about x axis, y axis and z axis respectively.

#### 2. Related Work

In the last five years, quadrotor has received tremendous number of contributes. Some contributes have been focused on the quadrotor structure [10], [11], [12] while the other contributes have been focused on the control algorithms; either by proposing a new control low or by comparing the performance of the others. One of these

control algorithms is based on the Linear Quadratic Regulator (LQR). LQR was implemented in several papers [13], [14]. This algorithm provides the best possible performance with respect to some given measurements. The main advantage of this algorithm is that the optimal input signal turns out to be obtainable from full state feedback.

Other control algorithms are done using an adaptive technique [15, 16]. The authors in [15] present a novel adaptive control algorithm using backstepping procedure in order to solve the problem of trajectory tracking for quadrotor aerial vehicles. They obtained the control law and then they tested it through numerical simulation. As stated above, this technique is usually used to handle uncertainties in the quadrotor model. Dynamic feedback and visual feedback techniques are also used in this field [17], [18], [19], [20], [21]. Dynamic feedback is implemented in several projects in order to transform the closed loop of the system into a linear and controllable subsystem. Visual feedback is also implemented using cameras. Instead of using onboard sensor information, this technique uses multi-cameras to present the position, velocity and attitude to the control system.

Finally, fuzzy techniques [22], and neural networks [23], [24] are also seen the light in this field. The authors in [22] propose a new adaptive fuzzy control to stabilize the quadrotor helicopter in the presence of sinusoidal wind disturbance. To solve the problem stated by the authors and to prevent the chatter in the control signal, the authors propose a set of alternate membership function as a guide to the adaptation process. In [23], the authors propose new adaptive neural network control to stabilize the quadrotor helicopter against modelling error and considerable wind disturbance.

### 3. Quadrotor's Kinematics and Dynamics

As shown in Fig.1, quadrotor operates in two coordinates system: inertial and body frame. Inertial reference frame located at the ground while the body frame located at the center of mass. The quadrotor has twelve governing state-variable forms in which they divided into four groups: Equations for the position of the vehicle, equations for the velocity of the center of gravity of the vehicle, equations of motion of the angular velocity of the vehicle and equations of angles of the vehicle with respect to the body frame. It should be noted that all the following derivations will be performed in the body frame coordinate. To find the four groups, the linear position of the center of mass of the quadrotor is determined by the coordinates of the vector between the origin of the body frame and the origin of the earth frame with respect to the inertial frame according to the equations:  $P = [x \ y \ z]^T$  and the vehicle's attitude described by  $A = [\Phi \ \Theta \ \psi \ ]^T$ , where  $\Phi$ ,  $\Theta$  and  $\psi$  denote the vehicle's roll, pitch, and yaw, respectively. The linear velocity of the quadrotor can be denoted with the following equation should be used.

$$\begin{array}{ccc}
x & u \\
y' &= R & v \\
z' & w
\end{array}$$
(1)

Where u, v and w being the absolute velocity components in the body frame and R is given as the airframe orientation in space and can be given by:  $R = R(\Phi) * R(\vartheta) * R(\psi) \cdot R(\Phi)$ ,  $R(\vartheta)$  and  $R(\psi)$  denote the rotation about X axis, Y axis, and Z axis respectively as shown in Fig.2.

	$cos  heta cos \psi$	$cos  heta sin \psi$	-sin heta
R =	$sin\psi sin heta cos\psi - cos arphi cos\psi$	$cos arphi cos \psi + sin  heta sin arphi sin \psi$	$sin \Phi cos  heta$
	$cos arPhi sin  heta cos \psi + sin arPhi sin \psi$	$sin  heta cos \Phi sin \psi - sin \Phi cos \psi$	cosΦcosθ

Using equation (1), it is possible to write;

(4)  

$$\mathbf{x}^{*} = (c\theta c\psi)\mathbf{u} + (-c\Phi s\psi + s\Phi s\psi s\theta)\mathbf{V} + (s\Phi s\psi + c\Phi s\theta c\psi)\mathbf{w}$$

$$\mathbf{y}^{*} = (c\theta s\psi)\mathbf{u} + (c\Phi c\psi + s\Phi s\psi s\theta)\mathbf{V} + (-s\Phi c\psi + c\Phi s\theta s\psi)\mathbf{w}$$
(5)  

$$\mathbf{z}^{*} = -s\theta \mathbf{u} + s\Phi c\theta \mathbf{v} + c\Phi c\theta \mathbf{w}$$
(6)

Where c = cos(), s = sin() and t = tan(). The main force acting on the quadrotor UAV is the gravity force which is simply:

F = ma, Where *m* is the total mass of the vehicle and *a* is the acceleration.

(7)  

$$Fx = mu` + m - rv + qw = m(u` - rv + qw)$$

$$Fy = mv` + m ru - pw = m(v` + ru - pw)$$
(8)  

$$Fz = mw` + m - qu + pv = m(w` - qu + pv)$$
(9)

*Fx, Fy* and *Fz* are the total external forces exerted on the center of mass along the *x, y, z* direction of the body frame. To find the second group, equation 7, 8, 9 are integrated.

(10)  

$$u^{`} = rv - qw - g \sin \theta + \frac{1}{m}(X + X)$$

$$v^{`} = pw - ru + g \sin \phi \cos \theta + \frac{1}{m}(Y + Y)$$
(11)  

$$w^{`} = qu - pv + g \cos \theta \cos(\phi) + \frac{1}{m}(Z + Z)$$
(12)

Where *p*, *q*, and *r* are the rate of rotation about the *x*, *y*, *z* axis (angular velocity components) of the body fixed frame. To find

(p, q, r).

(13)  
H = I. w  
(13)  

$$I = \begin{array}{cccc} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{array}$$
(14)

Where I is the inertia matrix, rearranging equation (14) results as;

$$I_{11}P^{`} = L + (I_{22} - I_{33})rq, \quad I_{22}q^{`} = M + (I_{33} - I_{11})rp, \quad I_{33}r^{`} = N + (I_{11} - I_{22})pq$$
(15)

Where L, M and N are the moments exerted on the vehicle about the x, y and z axes of the body frame.

Finally to evaluate the last group;

$$\begin{array}{c} \Phi^{`} \\ \theta^{`} \\ \psi^{`} \end{array} = Lib.w$$

$$\begin{array}{c} \psi^{`} \\ \end{array}$$
(16)

Where *Lib* is a rotation matrix from inertial frame to body frame components and it is given as;

0  $-sin\theta$ 1 Lib = 0 $cos\theta sin\Phi$ cos₽  $-sin\Phi$  cos $\theta$ cos $\Phi$ 0 (17) 0 ф` p1  $-sin\theta$  $\theta$ q = 0  $cos\Phi$  $cos\theta sin\Phi$  $r_B$ 0  $-sin\Phi$   $cos\theta cos\Phi$ ψì (18) Thus  $\Phi = p + qsin \Phi \tan \theta + r \cos(\Phi) \tan(\theta)$ (19)  $\theta = (\cos \Phi)q - (\sin \Phi)r$ (20)  $\psi = q \sin\phi \sec\theta + r \cos\phi s$ (21)

# 4. Linear Quadratic Regulator

Linear quadratic regulator (LQR) controller is considered as one of the most important state space based optimal control methods. In LQ problem, the system dynamics are described by a set of linear differential equations and its cost is described by a quadratic function. The solution for the problem is provided by the linear-quadratic regulator. For the derivation of the linear quadratic regulator, assume that the continuous-time linear system, defined on the interval  $t \in [t_0, t_1]$  is written in state-space form as:

a = Ax + Bu

(22)

The LQR controller is given by u = -Kx and the gain matrix K of the close loop system which solve the LQR problem is

$$K = R^{-1}B^T P$$
 (23)

So as to minimize the performance index

$$J = \int_{0}^{x} x \cdot Qx + u \cdot Ru \, dt$$

Where P is a unique, positive semi definite solution to the Riccati equation;

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$
(25)

## 4.1. LQR Design

To design the LQR controller, the first step is to select the weighting matrices *Q* and *R*, where Q is weighting factors that weight the states and R is also weighting factors that weights inputs. Then the feedback *K* can be computed and the closed loop system responses can be found by simulation.

# 4.2. Adaptive controller

As stated above, Model Reference Adaptive Control (MARC) is one type of adaptive control and it will be used in this paper. The Lyapunov stability argument is used to design the adaptive controller and the reference model used her for the controller is generated using LQR and quadrotor dynamics. The controller is formulated for the problem in the presence of parametric uncertainties in the form of missing the target location as a result of thrust variations.

From section III, equation of motion can be rewritten her by adding the terms that represent the uncertainties.

$$\dot{x_p} = A_m X_p + L_m U$$
(26)

Where (*Am*) is constant and unknown, *Lm* is unknown diagonal matrix. To track the position of the target *P* (*t*), the output error is defined by (e=y-p), where y is the system output and equal to ( $C^*Xp$ ). Adding the integrated output error to equation (26) we get

$$\dot{x_p} = A_m X_p + L_m U + L_m P$$
(27)

This equation will be used as a reference model to the MARC. The control input given by u = -Kx is added to the adaptive control input to give

$$U_a = Kx + \theta_p P + \theta_a = \theta^T . W^T$$
 (28)

Where  $\theta^{T}$ , and  $W^{T}$  are adaptive parameters that will be adjusted in the adaptive law given below

$$\theta = \quad \pounds \ W \ e^T \ P \ B$$
(29)

Where f is a diagonal matrix of adaptive gains, e is the model tracking error and it is the same as given in  $= R^{-1}B^TP^*$ . Thus the overall control input is

$$UT = U_a + U_n$$

(30)

Where Un is the input without uncertainties and Ua as given in  $U_a = Kx + \theta_p P + \theta_a = \theta^T$ .  $W^T$ . Thus, in the case with no parameter uncertainty, the input to the system is only that is given in u = -Kx while in the case that there is parameter uncertainty, the adaptive controller will assist the LQR in maintaining quadrotor in a stable mode.

#### 5. Simulation Results

In the following results, we set a reference point for the quadrotor to take off and fly over a target, keep it in level and then start to rotate at an angle of 30 degree (.523 radians). The reference point will be over the target with a state vector of x = [1 - 1 - 1] for x, y, z and with a state vector of  $O = [0 \ 0 \ .523]$  for  $\Phi$ ,  $\theta$  and  $\psi$  respectively. Keeping in mind that the quadrotor is starting from the ground with the initial position of  $x = [0 \ 0 \ 0]$ , hovering over the target, and keeping the pitch, roll angles and height constant.

Fig.3 shows the position and orientation of the quadrotor executing the simulated flight test in the case with no uncertainty. As we will see next, the uncertainty in this paper is represented in the form of loss of thrust (25%) initiated in one actuator at t = 28 s. The figure shows that the LQR controller is able to keep the quadrotor hovering stable over the target and following the reference point.



Fig.3. Time response of LQR controller with no uncertainty.

Fig.4 shows a comparison between LQR controller and the adaptive controller; wherein Fig.4 (a-b) shows position and orientation of the LQR controller in the case of uncertainty while Fig.4 (c-d) shows position and orientation of the adaptive controller in the case of uncertainty.







Fig.4 Comparison between LQR controller and adaptive controller.

As shown in the figure, LQR experiences a deviation of 25cm in the position and over seven degrees in yaw for the quadrotor under test. This deviation causes the quadrotor to fly at altitude of 75 cm during the loss of 25% of thrust and then return back to the normal situation. On the other hand, the adaptive controller has a smoother response than the LQR controller during the loss of thrust. In addition, it responses quickly and keeps the variation less than 5cm during the failure.

#### 6. Conclusion and Future Work

Quadrotor is a small aircraft whose lift is generated by four rotors. It can move in any direction and is capable of hovering and fly at low speed. In this paper, we present a new adaptive control system to stabilize the quadrotor that is designed to perform a variety of specific tasks. A step by step derivation of the kinematics and dynamics of the quadrotor are derived, and then the baseline controller is redesigned in the case where there is an uncertainty in finding targets in search and rescue missions. Moreover, a new term is added to the feedback linearization controller in order to guarantee that the vehicle will not miss the target location and to offer robustness to the system in the presence of parametric uncertainties. The simulation results show that the proposed controller is able to handle the uncertainty behaviour in hazardous environments. The next step is to test the proposed controller on a real quadrotor built specifically for this paper.

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